Stochastic Modeling of The Decay Dynamics of Online Social Networks

Mohammed Abufouda and Katharina Anna Zweig, Computer Science Department, University of Kaiserslautern, Germany

Complenet 2017, Dubrovnic, Croatia

Dynamics of networks:

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• Growth dynamics

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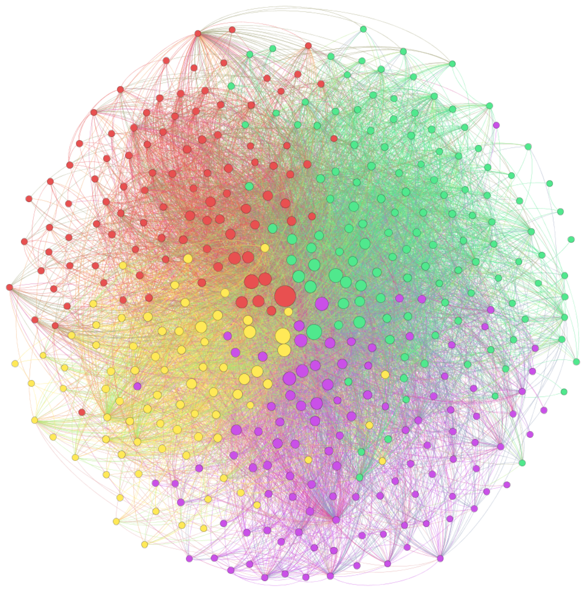

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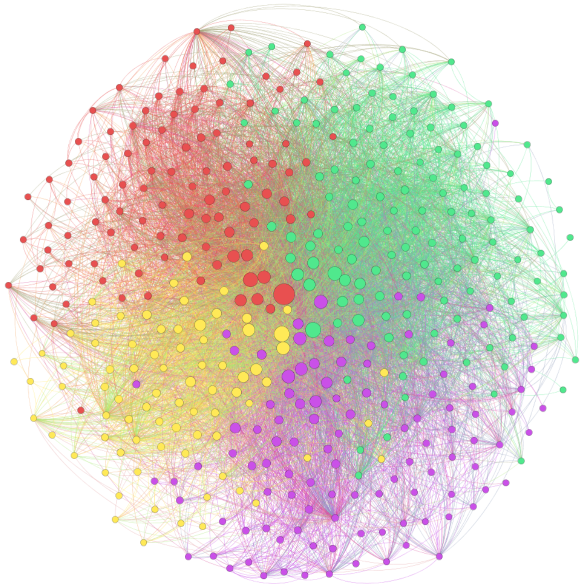

The decay of Startup Business website
Oct-2009 to Nov-2011

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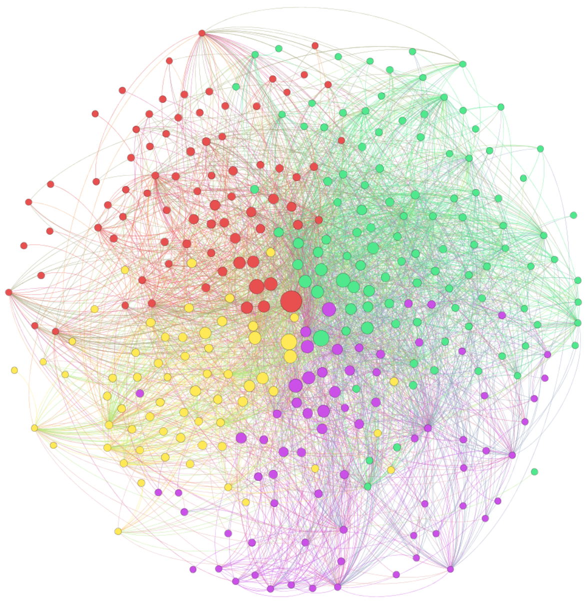

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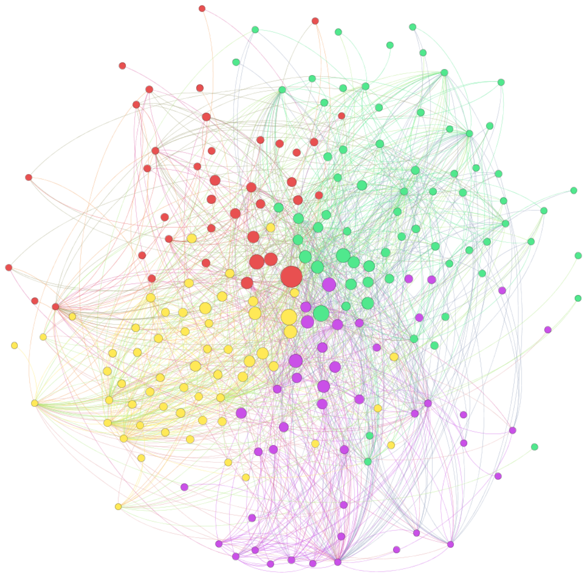

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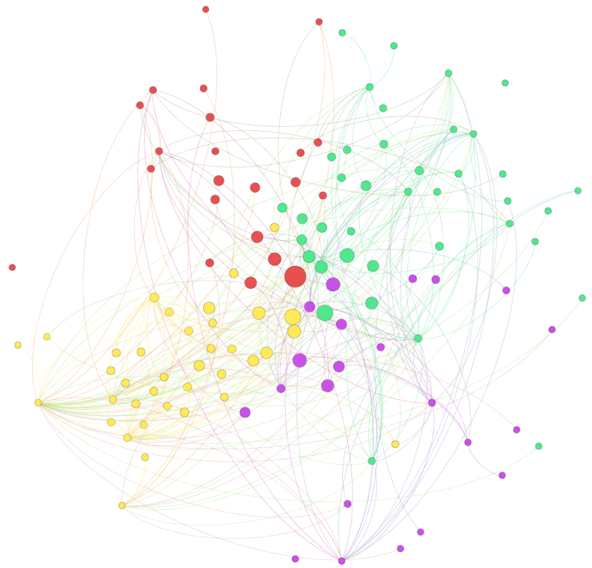

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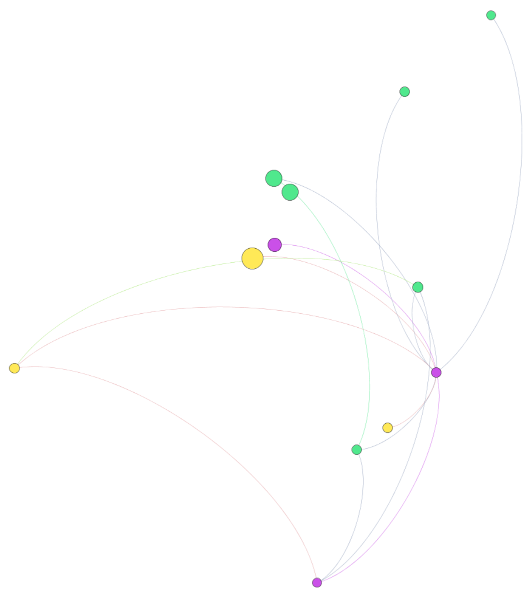

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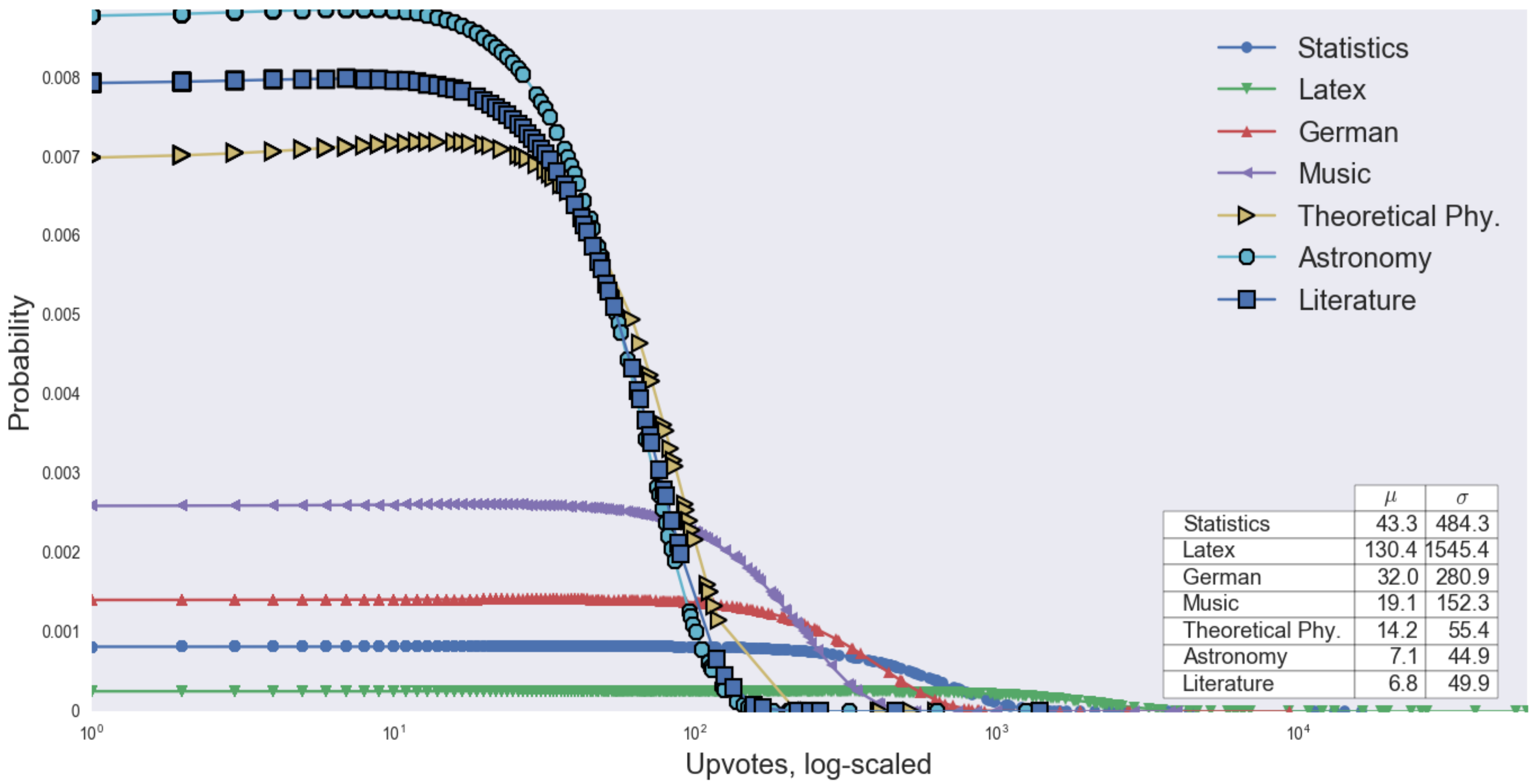
Probability
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
10^0
10^1
10^2
10^3
10^4
Upvotes, log-scaled
Statistics
Latex
German
Music
Theoretical Phy.
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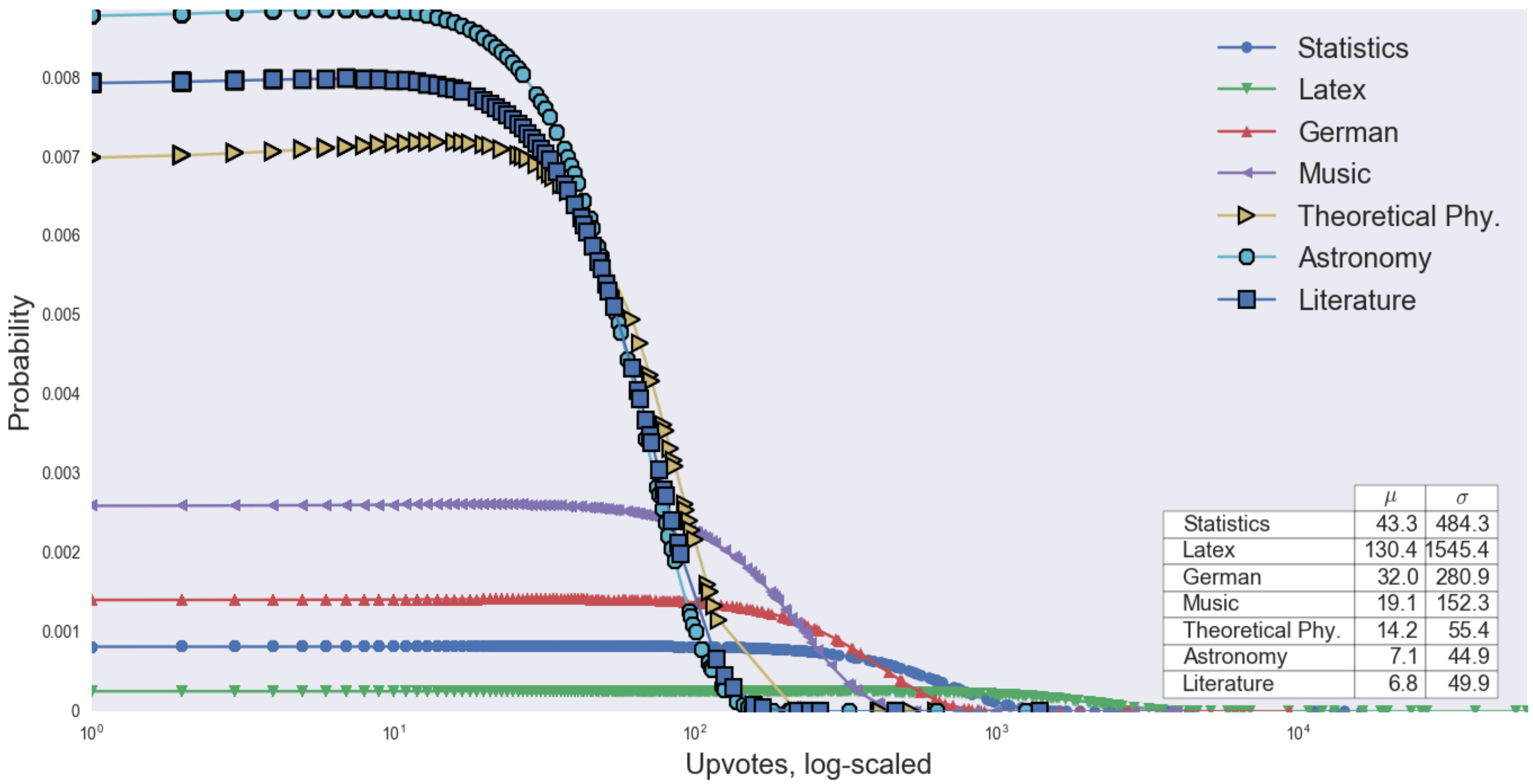
Literature
	μ	σ
Statistics	43.3	484.3
Latex	130.4	1545.4
German	32.0	280.9
Music	19.1	152.3
Theoretical Phy.	14.2	55.4
Astronomy	7.1	44.9
Literature	6.8	49.9



We found the same difference for many types of interactions like comments, reputation…etc.

The model & Assumptions

- Each node has an initial leave probability $\pi_v^{t=0}$
- The leave probability increases over time

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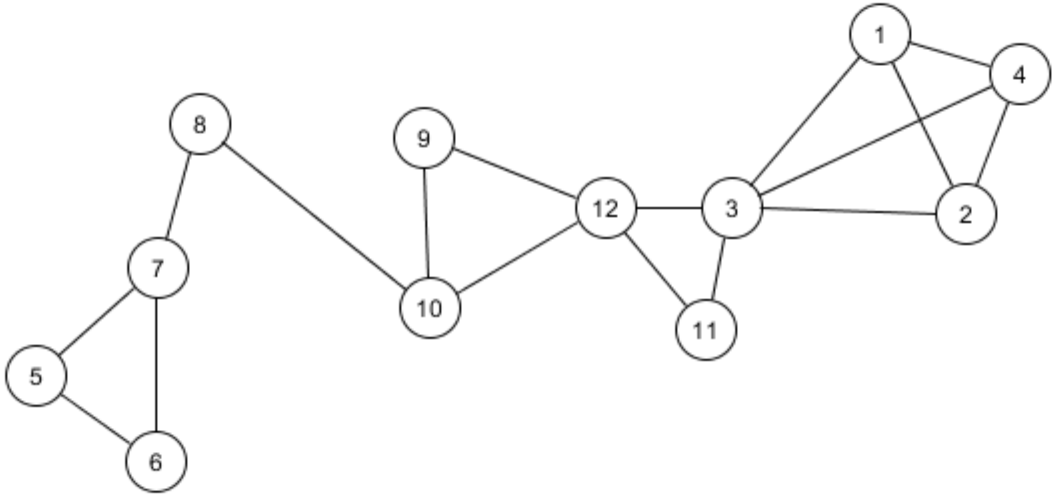

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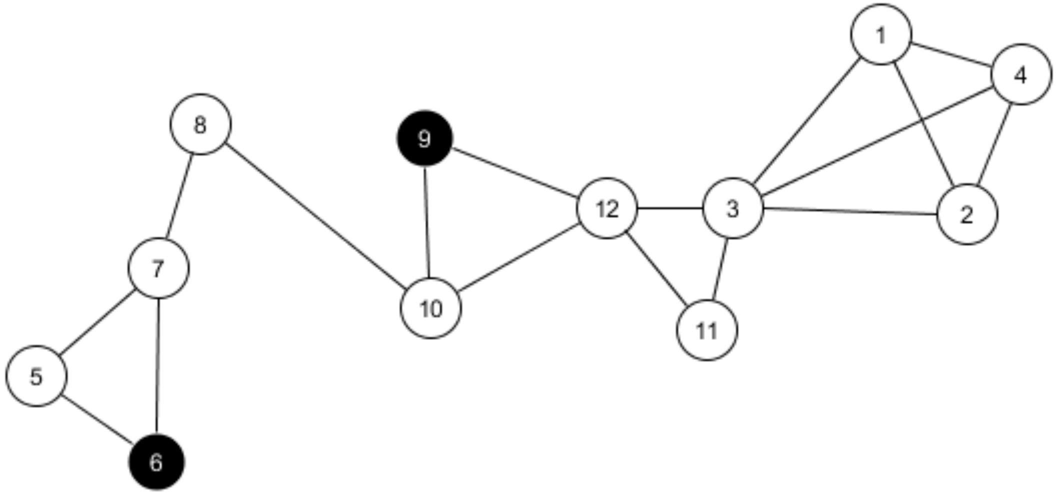

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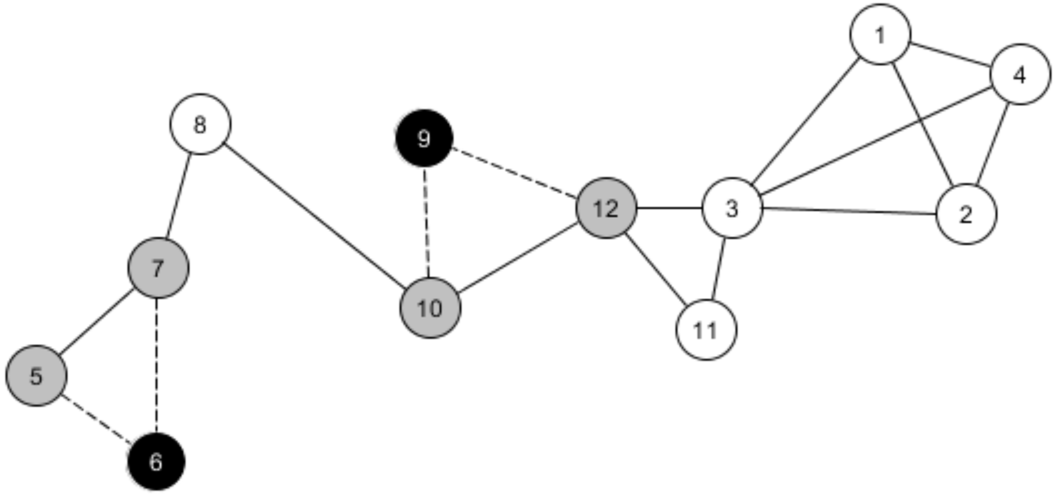

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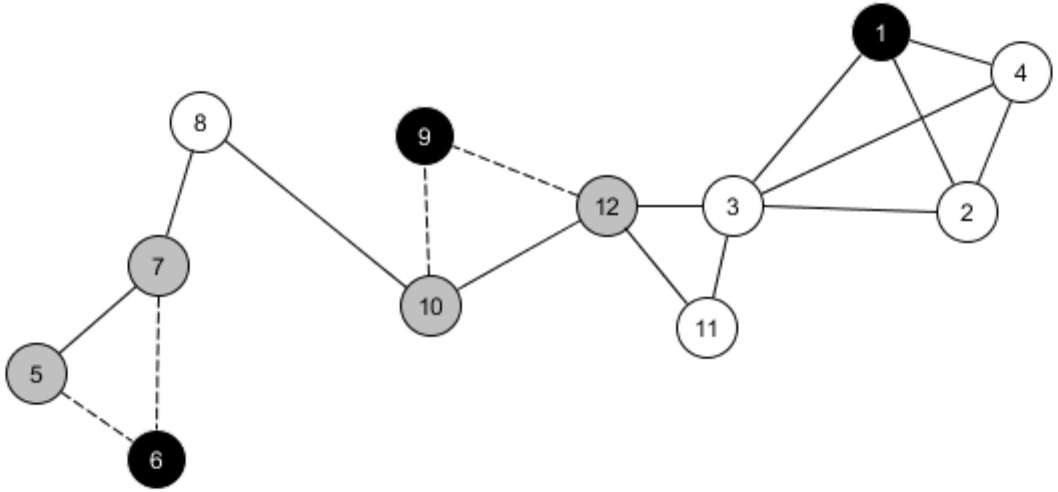

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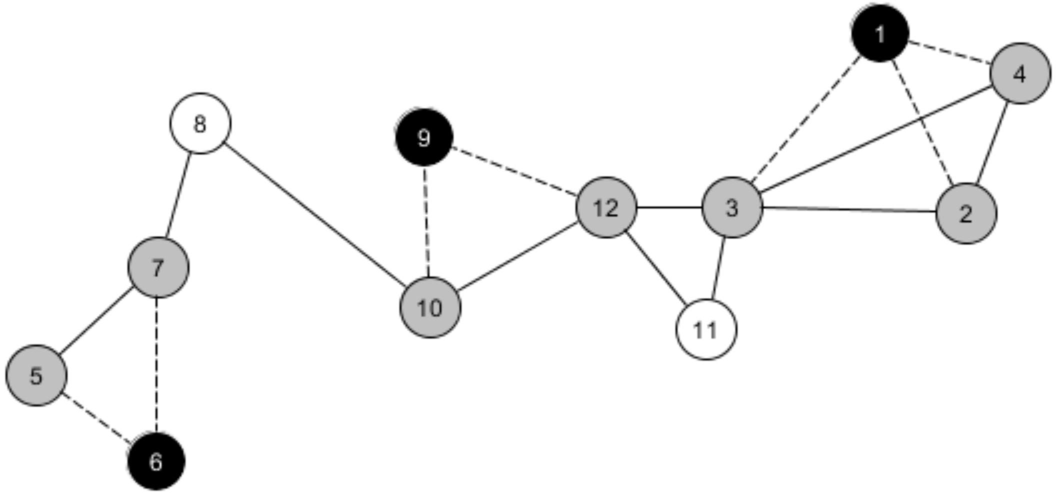

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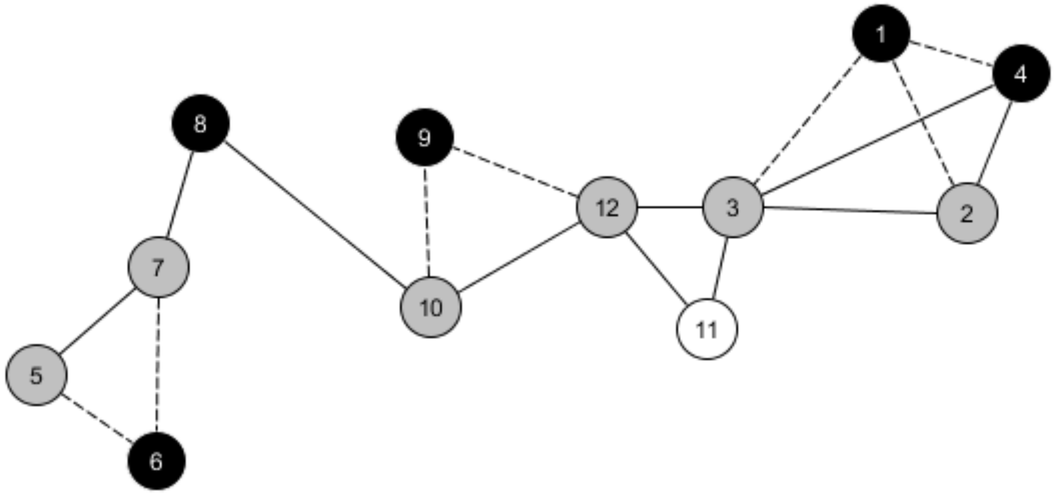

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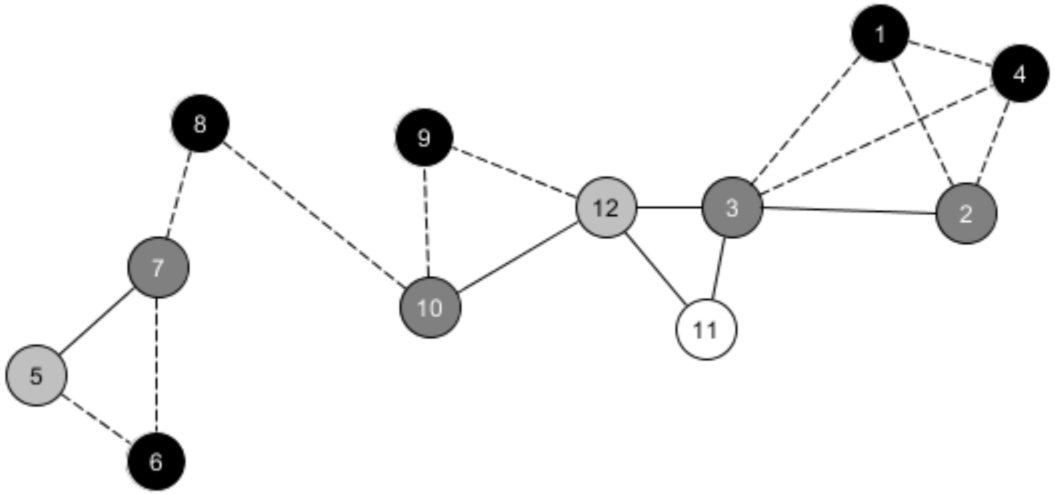

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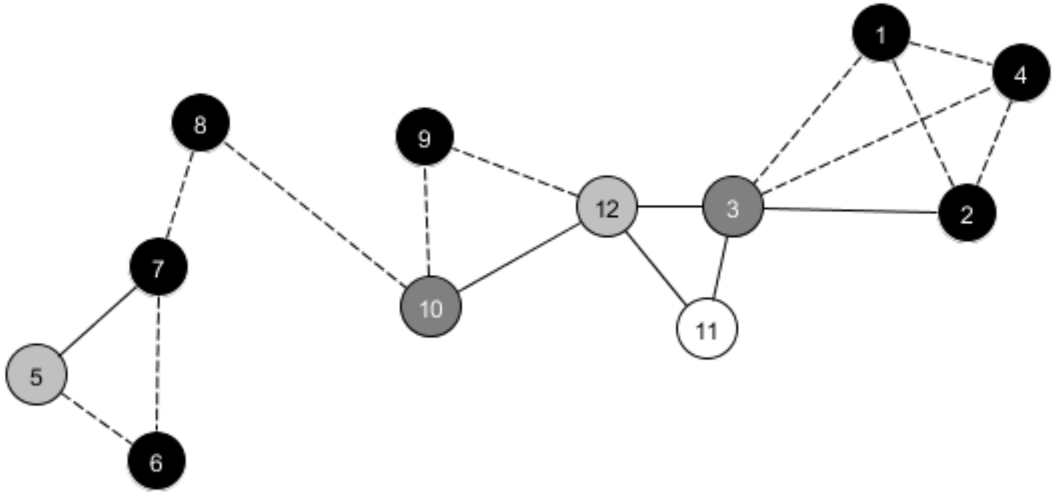

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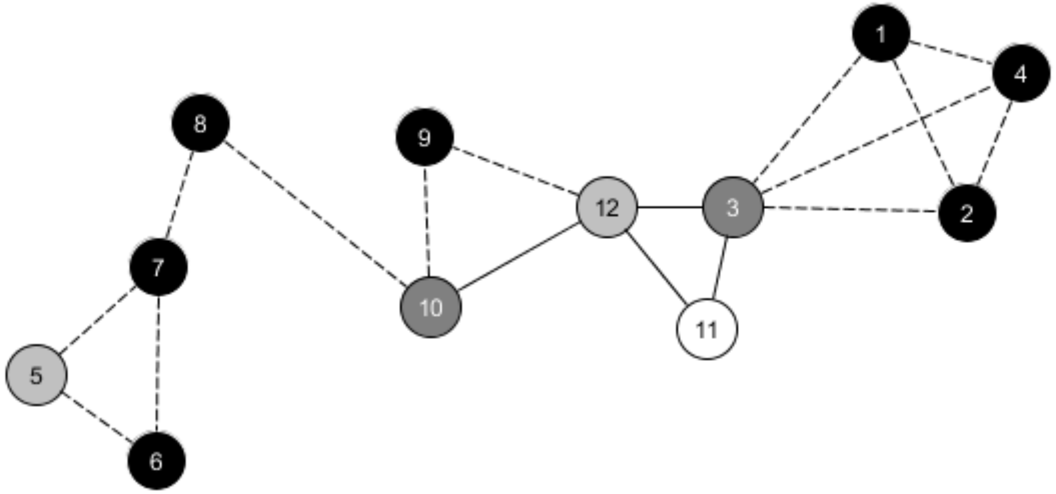

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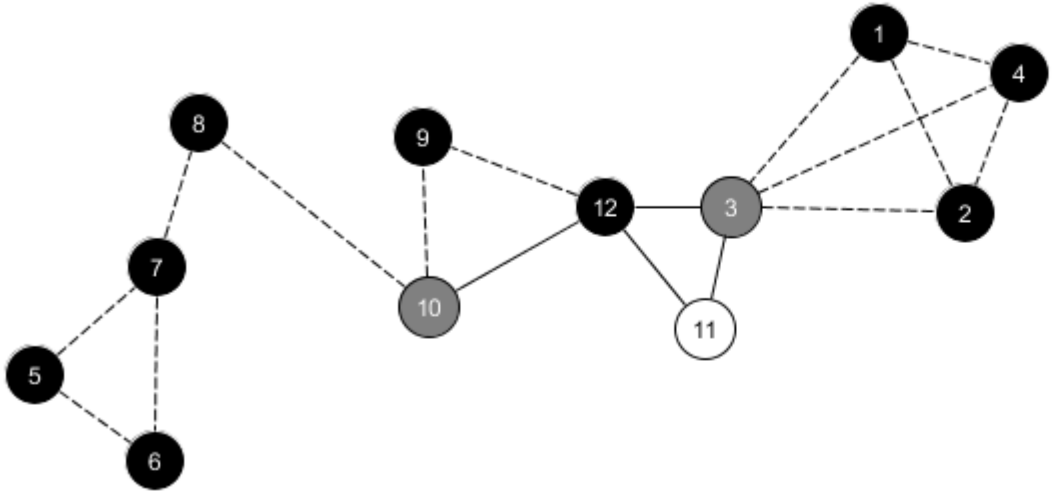

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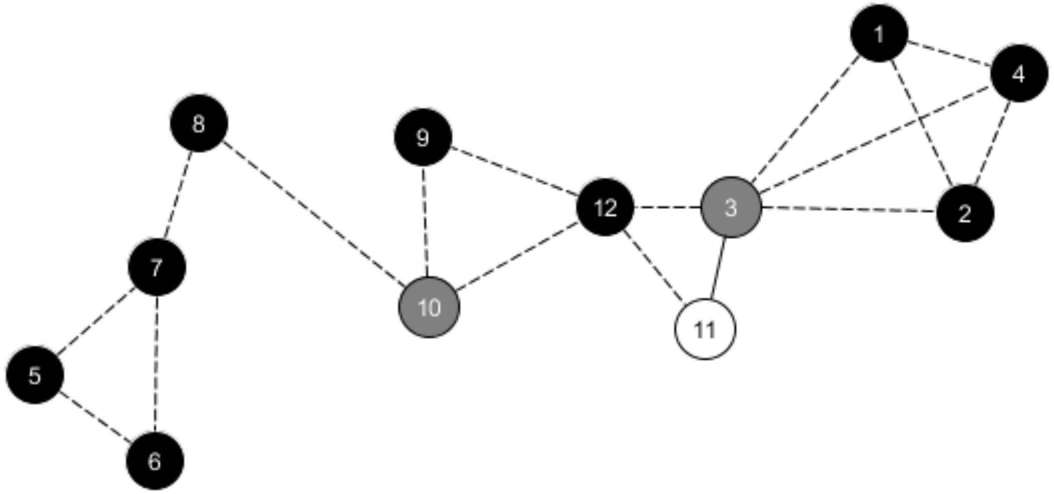

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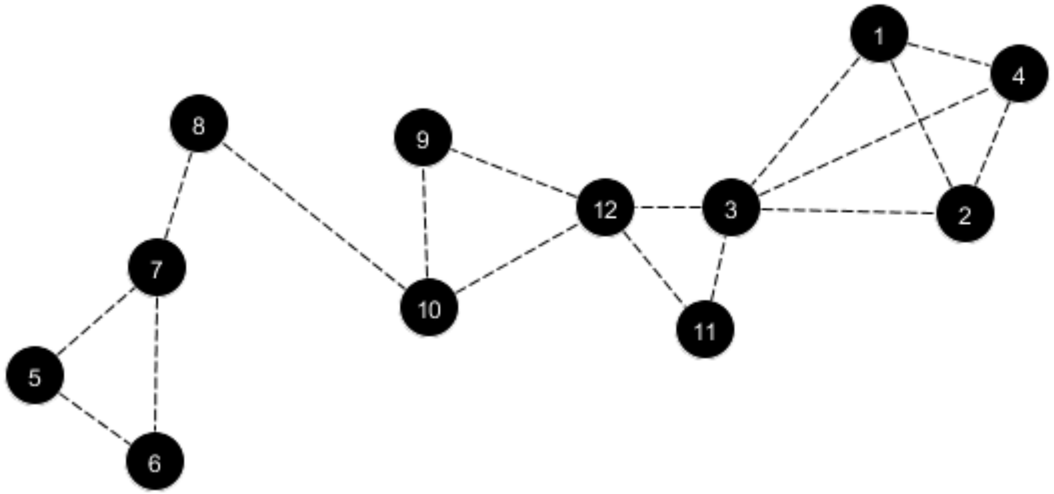

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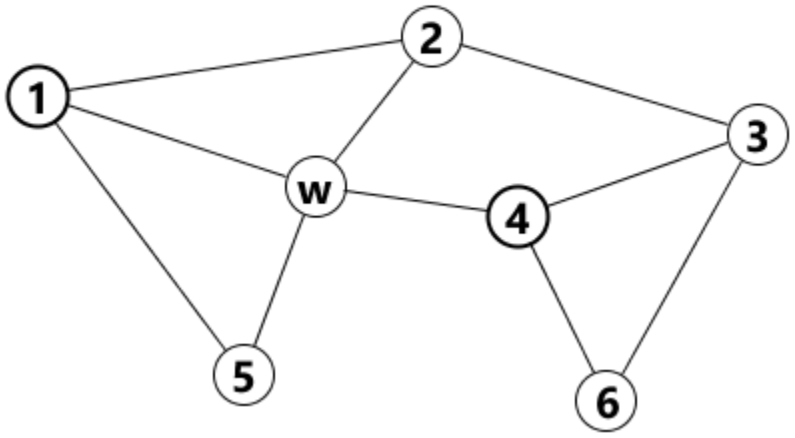

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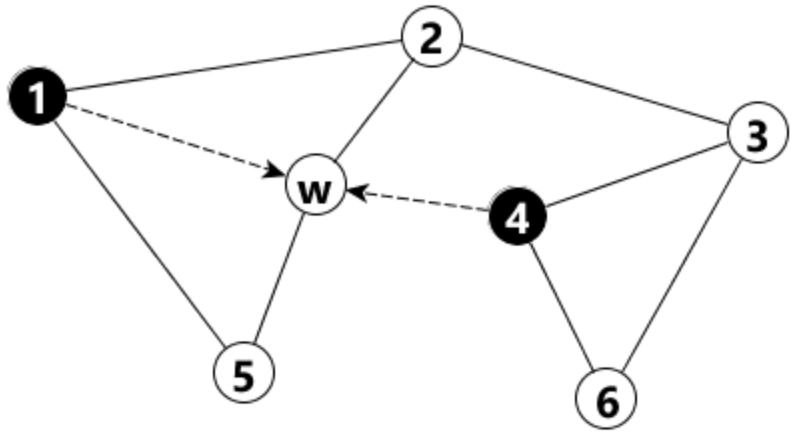

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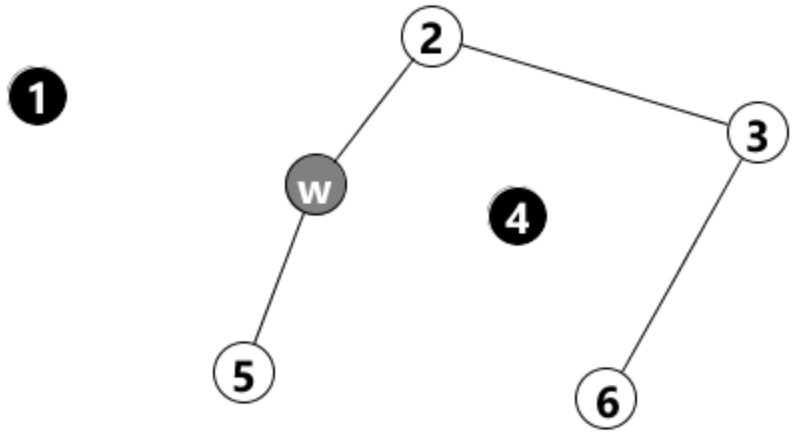

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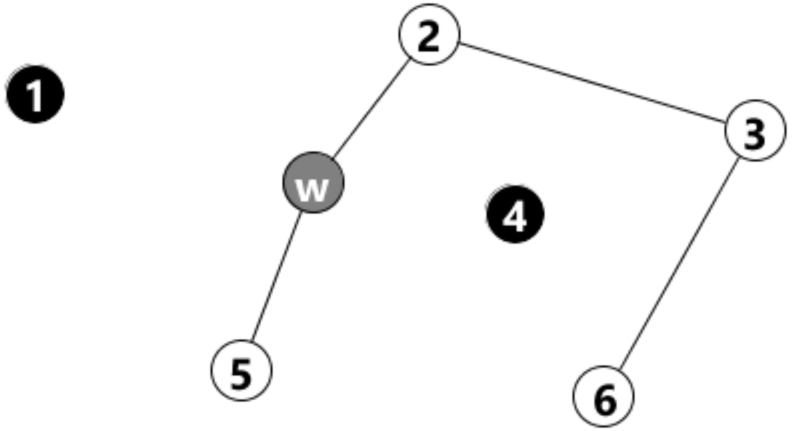

$$\Delta\pi_w^t = 1 - [\underbrace{(1 - \xi_w^{t-1})}_{\text{Assures leave}} \underbrace{(\prod_{u \in \overline{\Gamma}_w^{t-1}} (1 - \pi_u^{t-1}))}_{\text{Leave probabilities effect}} \underbrace{(\prod_{u \in \overline{\Gamma}_w^{t-1}} (1 - \delta_{u,w}^{t-1}))}_{\text{Tie strength effect}}]$$

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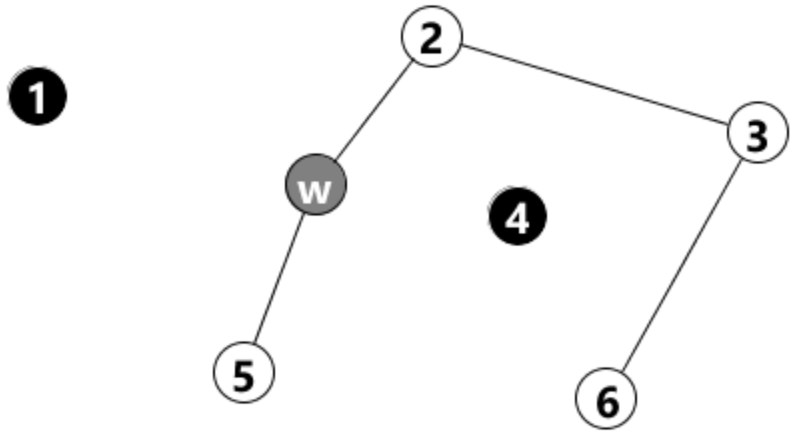

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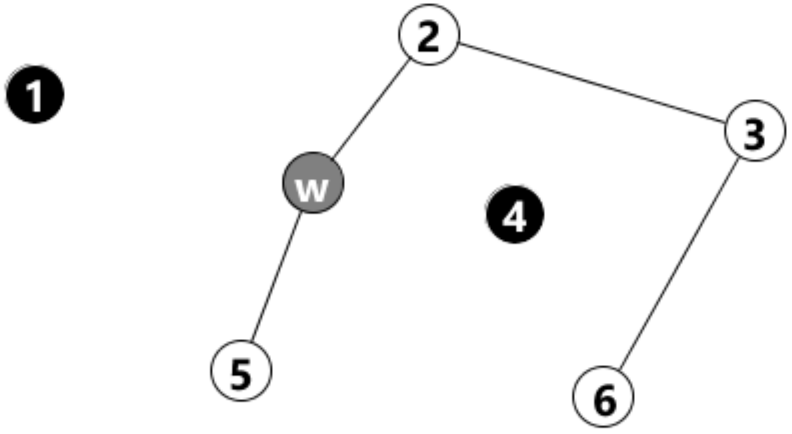

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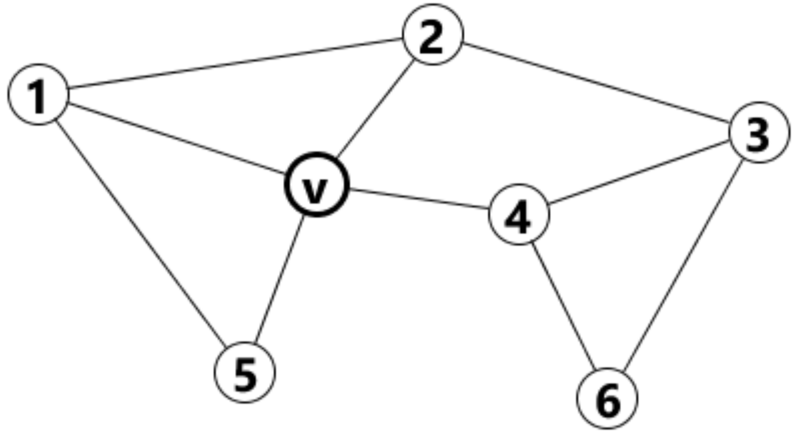

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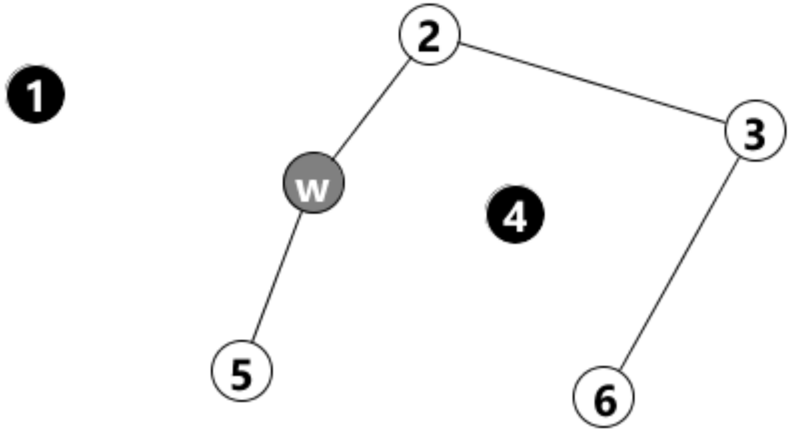

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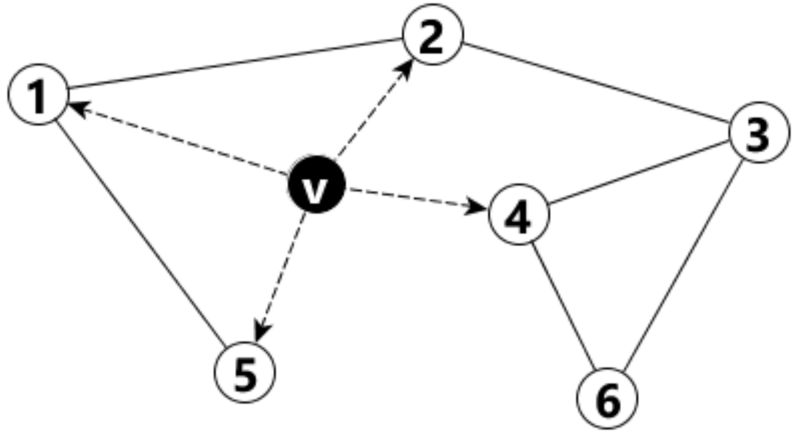

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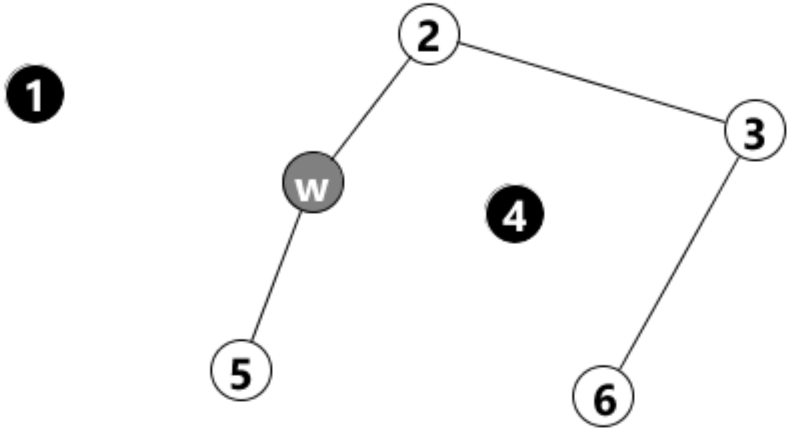

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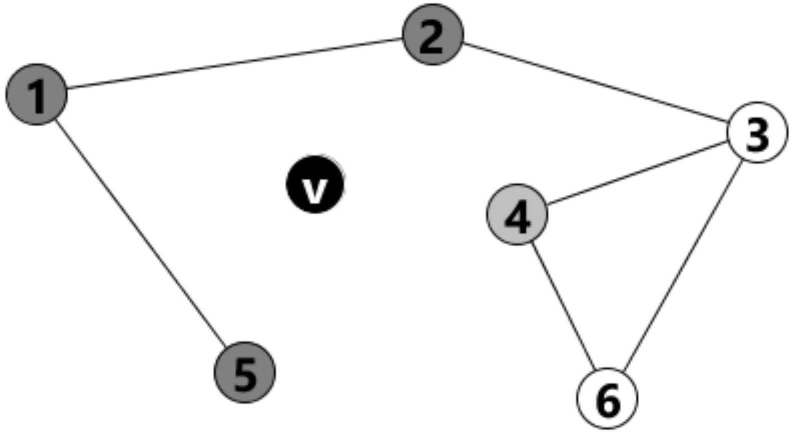

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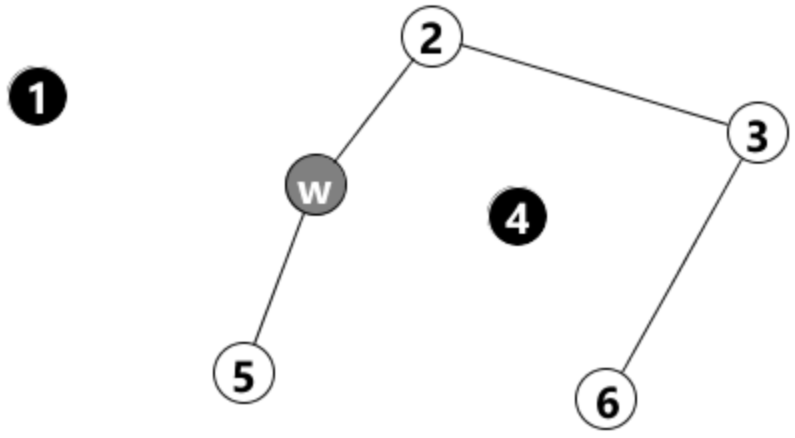

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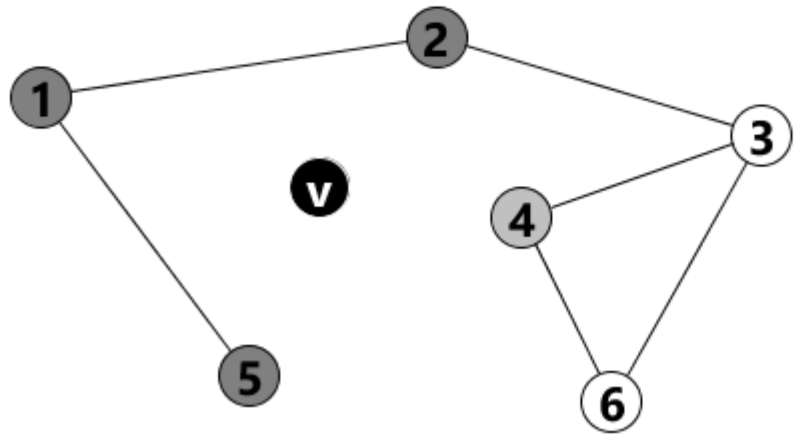

$$\Delta\pi^t(v) = \sum_{w\in\underline{\Gamma}_v^{t-1}} 1-(1-\pi_v^{t-1})(1-\delta_{v,w}^{t-1})$$

Theoretical results

- Theorem 1: The node leave equation is *submodular*
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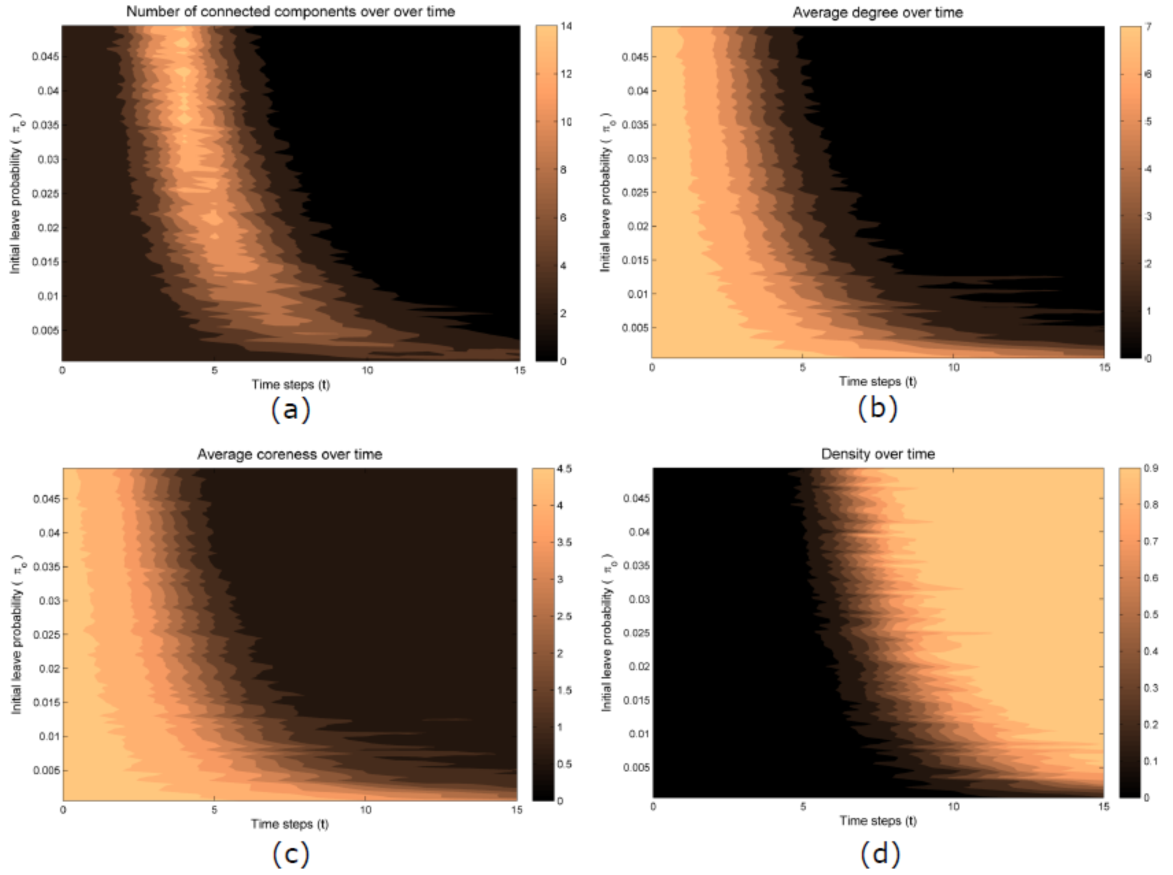

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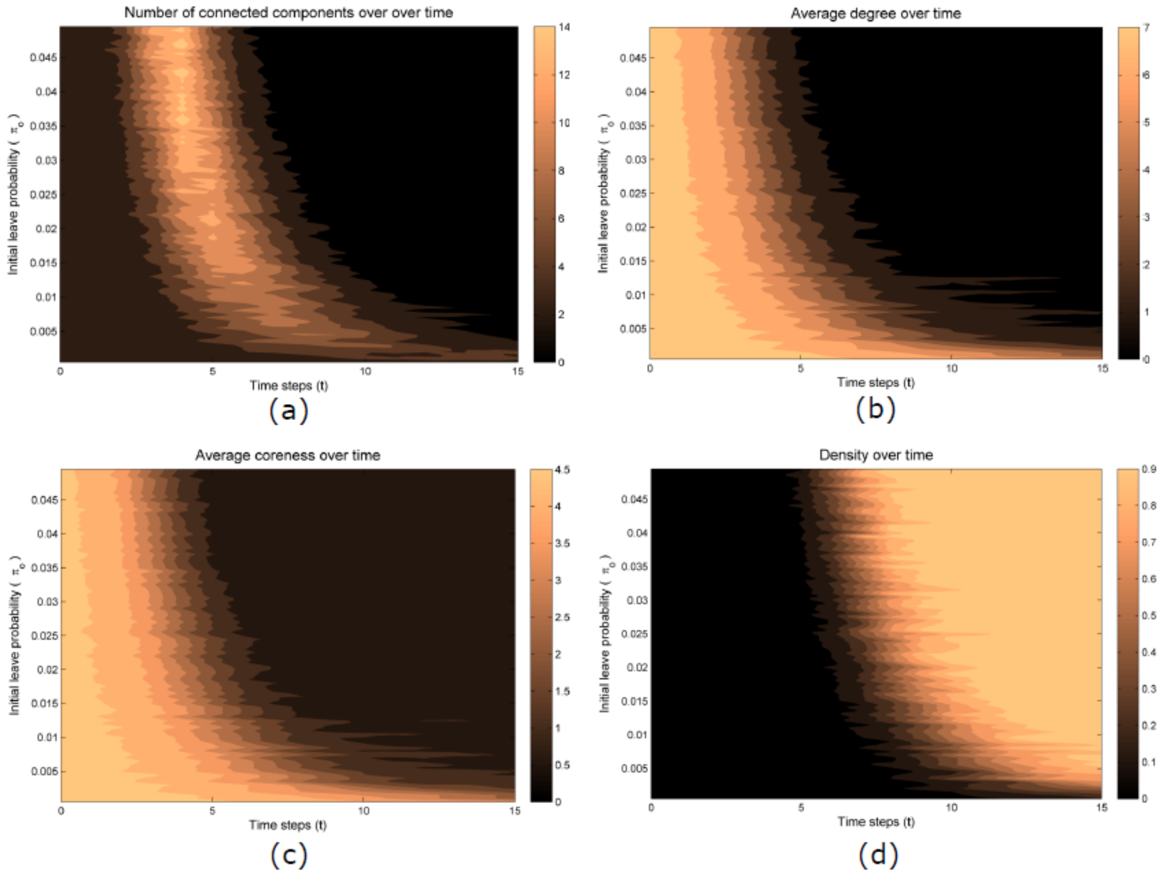

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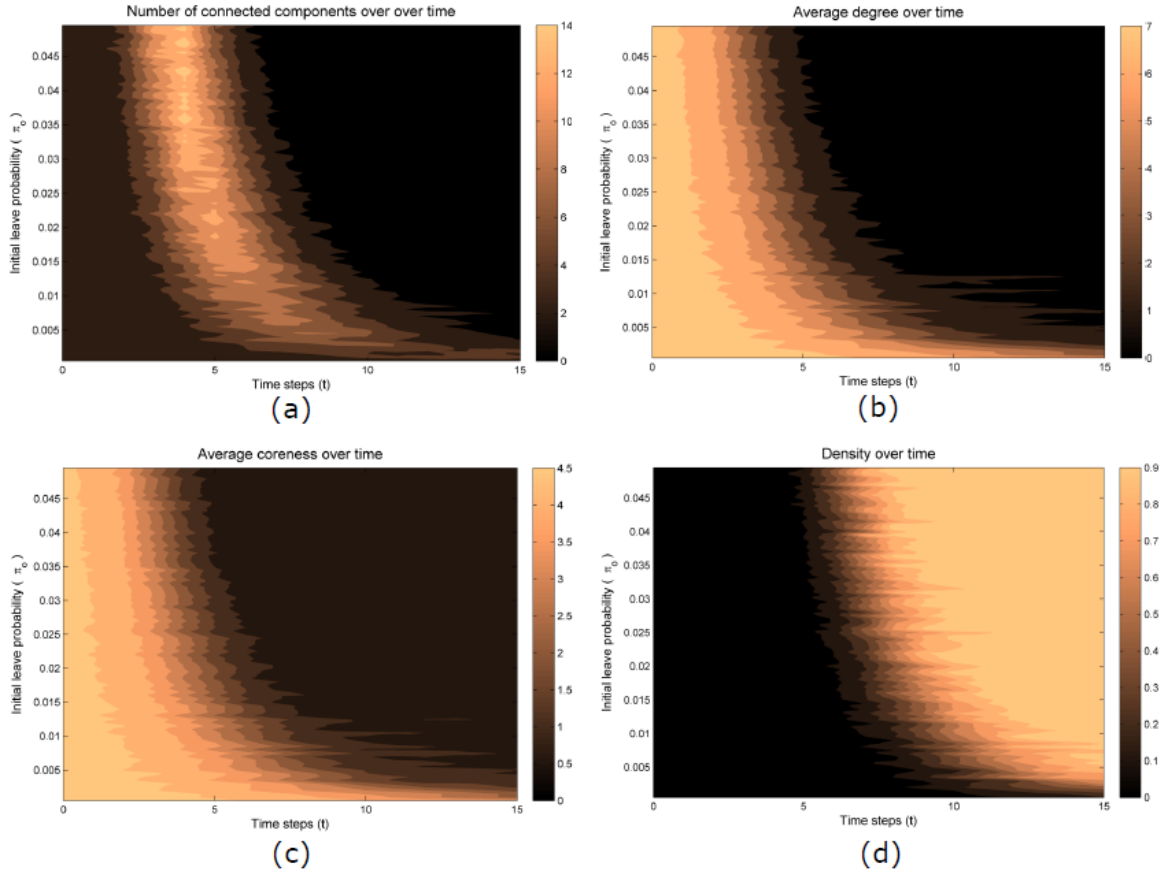

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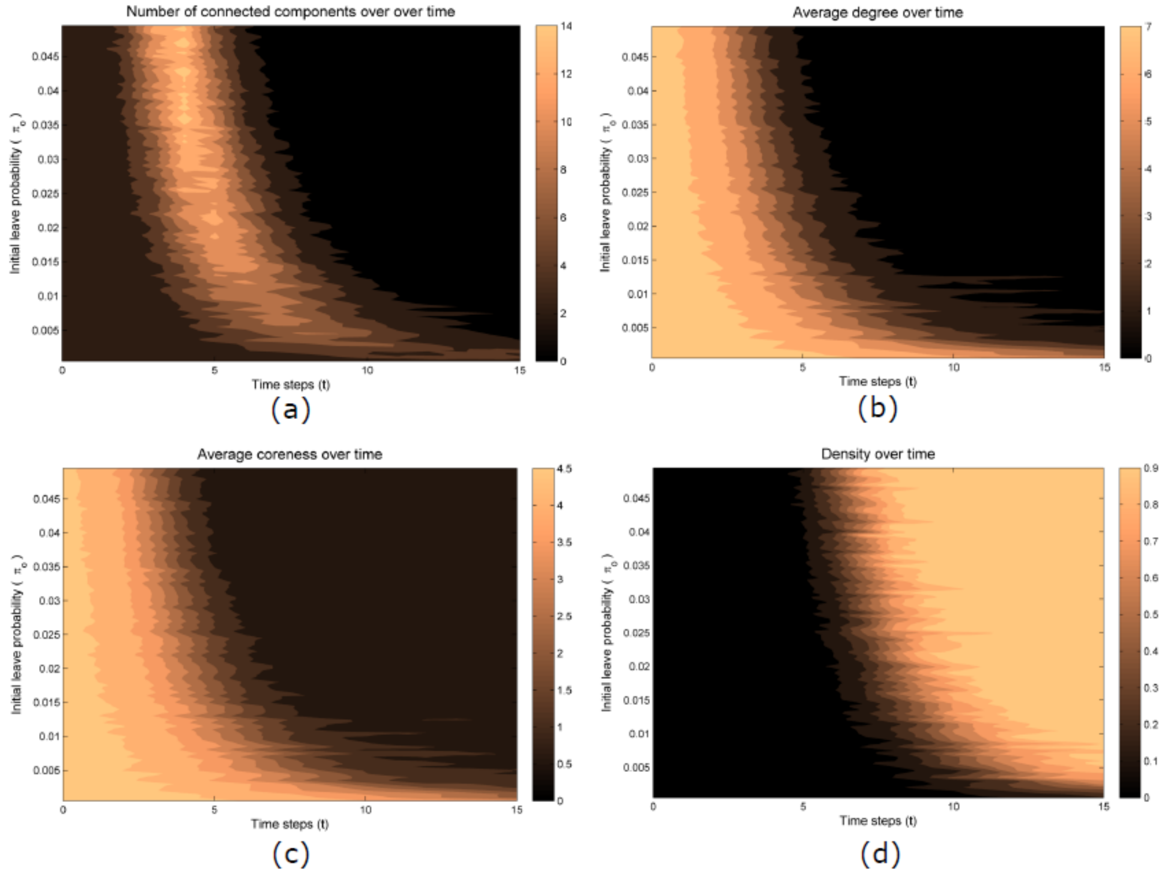

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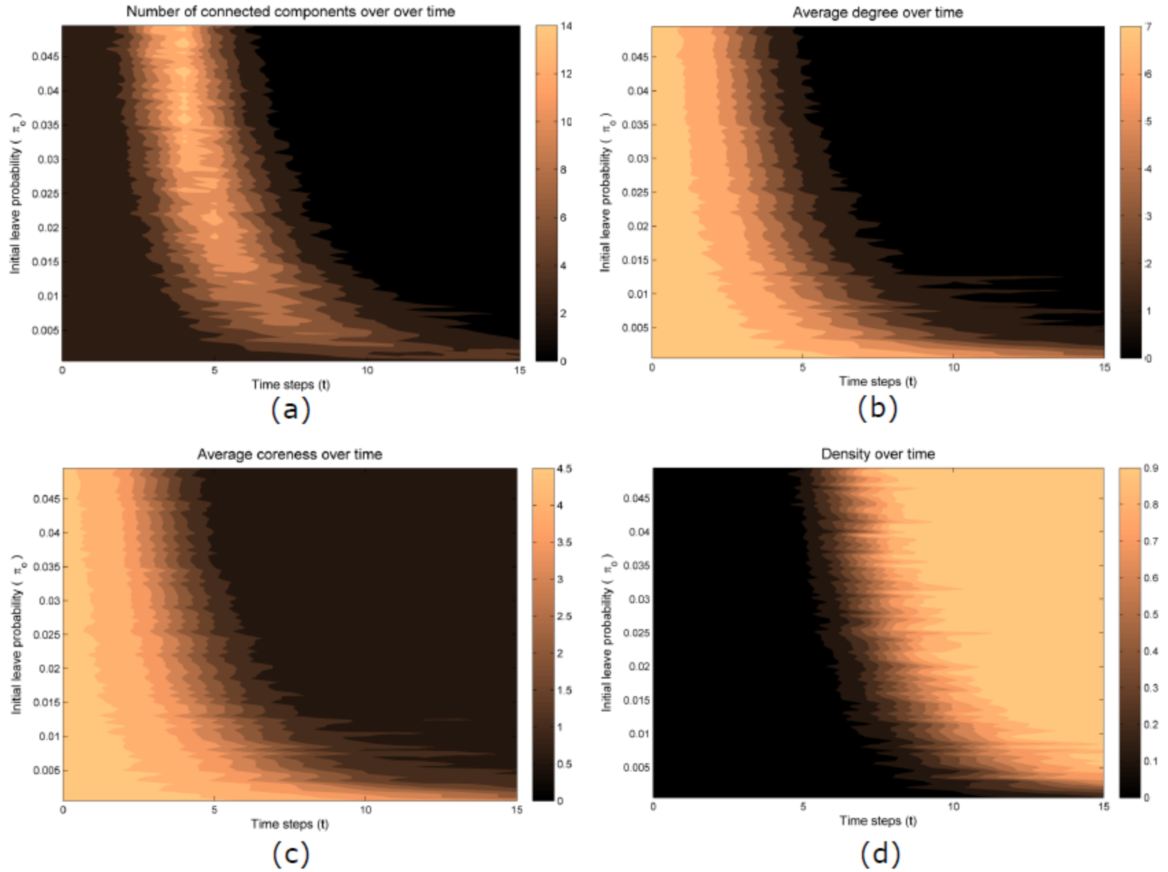

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- Maximizing the leave effect
- Engineering resilient networks